

## Eton College King's Scholarship Examination 2009

(One and a half hours)

### MATHEMATICS A

*Answer Question 1 and as many of the other five questions as you can.*

*Question 1 is worth 50 marks. All other questions are worth 10 marks each.*

*Show all of your working. The use of calculators is permitted.*

#### 1. Compulsory Question

(a) Solve the following equations:

(i)  $3(x+1) - 2 = 16$

(ii)  $\frac{6x-2}{5} - \frac{5x-3}{7} = 1$  [2, 2]

(b) Sachin scored 75 in one part of an exam, which was out of 200, and scored 60% in the second part of the exam, which was out of 50.

(i) What was his percentage score in the first part of the exam?

(ii) What was his mark in the second part of the exam?

(iii) What was his overall percentage in the exam?

[1, 1, 2]

(c) On a January morning the temperature in Windsor was  $-1^{\circ}\text{C}$ . In the evening it had fallen by  $3^{\circ}\text{C}$ .

(i) What was the temperature in the evening (in Celsius)? [1]

If  $c$  is the temperature in Celsius then the temperature,  $f$ , in Fahrenheit is given by the formula  $f = \frac{9}{5}c + 32$ .

(ii) Find the difference (in Fahrenheit) between the morning and evening temperatures in Windsor. [2]

(iii) Make  $c$  the subject of the equation  $f = \frac{9}{5}c + 32$  giving your answer in the form  $c = pf + q$  where  $p$  and  $q$  are fractions. [2]

(iv) The following week the temperature was  $41^{\circ}\text{F}$ . Find this in Celsius. [1]

(v) What temperature is the same in Celsius and in Fahrenheit? [2]

(d) (i) Solve the inequality  $\frac{3x-5}{2} > 14$ . [2]

(ii) Represent the solution to the inequality  $12 - 5x > -8$  on a number line. [3]

- (e) Copy the following calculations, adding brackets to make them correct:
- (i)  $3 + 2 \times 4 + 2 \times 3 = 90$  [2]
- (ii)  $5 \times 2 + 7 - 1 = 2 + 3 \times 8 + 6$  [3]
- (f) A bottle of car cleaning solution states that you should add 15ml of cleaning solution to a bucket containing 5 litres of water.  
The bottle contains 600ml of cleaning solution.
- (i) How many ml of cleaning solution must be added to a bucket containing 4 litres of water? [2]
- (ii) Peter reckons he needs 8 litres of water to wash his car properly. He washes his car every 6 weeks. How many weeks will the cleaning solution last? [2]
- (g) A website has an offer in which all individual songs cost a set price and all albums cost a set price. Joshua can't remember how much the site charged for individual songs and albums but he knows that two individual songs and three albums cost him £19.55 and that three individual songs and four albums cost him £26.33.
- If £ $s$  is the price of an individual song and £ $a$  is the price of an album then:
- (i) Write down two equations involving  $s$  and  $a$ . [2]
- (ii) Solve these to find  $s$  and  $a$ . [4]
- (h) A triangle has sides  $3x - 2$ ,  $8x$  and  $9x - 2$ . Its perimeter is 56cm.
- (i) Find the value of  $x$ . [2]
- (ii) State, with clear reasons, whether or not the triangle is a right-angled triangle. [2]
- (i) Express the following in the form  $\frac{m}{n}$ . Show all your working:
- (i)  $3\frac{1}{2} + \frac{2}{7}$  [2]
- (ii)  $\frac{a}{b} + \frac{b}{a}$  [3]
- (j) In a bag there are 3 blue balls, 5 red balls and  $n$  yellow balls.
- (i) Find, in terms of  $n$ , the probability of a ball, chosen at random, being yellow? [2]
- (ii) If the probability of choosing a yellow ball is  $\frac{1}{3}$  then find  $n$ . [3]

Please turn over

2. A 2x2 grid is placed on this 8x8 grid so that it consists of four numbers as shown below:

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

$S(k)$  is the sum of the 4 numbers in the 2x2 grid in which  $k$  is the smallest number. So from above we see that  $S(11) = 11 + 12 + 19 + 20 = 62$ .

- (a) Find  $S(45)$ . [1]
- (b) Copy and complete this grid, writing each of the three missing numbers in terms of  $k$ .

$k$	...
...	...

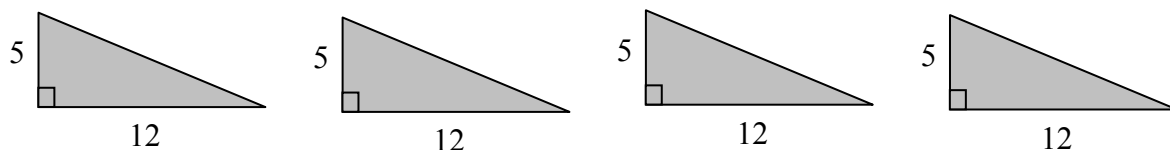
- (c) Find an expression for  $S(k)$  in terms of  $k$ . [2]
- (d) Is it possible to find a value of  $k$  such that  $S(k)$  is a multiple of 4? [1]

$P(k)$  is the product of the top right and the bottom left numbers in the 2x2 grid in which  $k$  is the smallest number. So from above we see that  $P(11) = 12 \times 19 = 228$ .

$Q(k)$  is the product of the top left and the bottom right numbers in the 2x2 grid in which  $k$  is the smallest number. So from above we see that  $Q(11) = 11 \times 20 = 220$ .

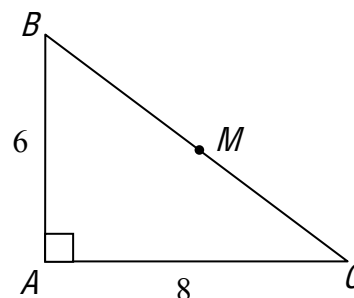
- (e) Use (b) to find an expression for  $P(k) - Q(k)$  showing all your working clearly. [3]
- (f) Find  $P(k) - Q(k)$  if the 2x2 grid was placed on a 10x10 grid. [1]

3. (a) The four right-angled triangles shown below with width 12 and height 5 are cut out of a piece of cardboard.



Use a diagram to show how these four triangles can be put together to form one large right-angled triangle, stating clearly the lengths of its three sides. [2]

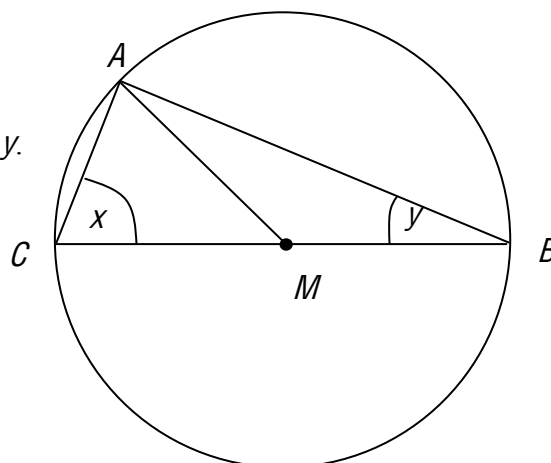
- (b) (i) In the right-angled triangle  $ABC$  shown,  $AC$  has length 8,  $AB$  has length 6 and  $M$  is the midpoint of  $BC$ .



By considering the reverse process of (a) or otherwise, calculate the length  $AM$ . [2]

- (ii) Suppose a circle was drawn with centre  $M$  such that the circle passed through  $A$ . Through which other point or points does the circle pass? [1]

- (c) The circle shown has centre  $M$  and passes through the points  $A$ ,  $B$  and  $C$ .  $BC$  is a diameter of the circle. The angle  $MCA$  is  $x$  and the angle  $MBA$  is  $y$ .



- (i) Find (in terms of either  $x$  or  $y$  or both) the angles  $CAM$  and  $MAB$ . [2]  
 (ii) By considering the angles in the triangle  $ABC$ , find the value of  $2x + 2y$ . [1]  
 (iii) Hence find the value of the angle  $CAB$ . [2]

4. (a) Through how many degrees (in terms of  $m$ ) does the minute hand of a clock rotate in  $m$  minutes where  $m < 60$ ? [2]
- (b) Through how many degrees (in terms of  $m$ ) does the hour hand rotate in  $m$  minutes where  $m < 60$ ? [2]
- (c) What is the obtuse angle in degrees between the hands at 0920? [2]
- (d) At what time between 1130 and 1200 is the angle between the minute hand and the hour hand equal to  $44^\circ$ ? [4]



5. A certain country has an inflation rate of 10%. This means that the price of, for example, a loaf of bread increases by the same proportion each month so that in a year, the price of a loaf of bread has increased by 10%. Suppose that this inflation rate was constant over 2007 and 2008 and that a loaf of bread costs 1000 Florins at the start of 2007.
- (a) How much would a loaf of bread have cost at the end of 2007? [1]
- (b) How much would a loaf of bread have cost at the end of 2008? [2]
- (c) How much (to 1dp) would a loaf of bread have cost half way through 2007? [2]

In October of last year this was a headline on the *Guardian* website:

Zimbabwe's inflation rate surges to 231,000,000%

Chris McGreal, Africa correspondent  
guardian.co.uk, Thursday October 09 2008 15.29 BST  
[Article history](#)

- (d) If this rate of inflation remained constant then by what factor would you have to multiply prices from October 2008 to find prices in October 2009? [2]

Suppose that on Thursday 9th October 2008, this extraordinary rate of inflation meant that a loaf of bread cost \$10 million.

- (e) By treating 3 months as being exactly one quarter of a year, calculate how much the loaf of bread would have cost 3 months on from this date? [3]

6. A sphere of radius  $r$  has a surface area of  $4\pi r^2$  and a volume of  $\frac{4}{3}\pi r^3$ .

One winter's day, Alex and Bertie create large spherical snowballs for making a snowman.

Alex's snowball has a radius of 36cm whilst Bertie's snowball has a radius of 15cm.



- (a) Calculate the radius of a spherical snowball which has the same surface area as the combined surface area of Alex and Bertie's snowballs. [2]

Bertie now makes his snowball bigger so that it has a radius of 18cm. He then places his snowball on top of Alex's. They then place both snowballs in a cylindrical container and wait for them to melt completely.

The density of each snowball is two-thirds of the density of water.  
So  $3\text{ cm}^3$  of snow in the snowball melts to give  $2\text{ cm}^3$  of water.

- (b) If the water reaches a height of 16cm in the cylindrical container when the snowballs have both melted completely then find the radius of the cylindrical container. [3]
- (c) Express the total volume of the water in the cylinder in form  $\pi \times 2^p \times 3^q$  where  $p$  and  $q$  are whole numbers. [2]
- (d) Hence find any other whole number values for the radius of the cylinder such that the height of water in the cylinder is also a whole number. Remember that the radius must be at least 36cm for the snowballs to fit in! [3]

[End of Paper]