

RADLEY COLLEGE
Entrance Scholarships



MATHEMATICS II

Thursday 8th March 2001

Time allowed 2 hours

*You may try the questions in any order and
you are not expected to complete them all.*

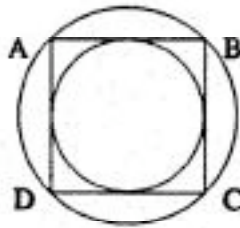
Show all working.

1. Ahmed would like to buy a carpet. If he pays cash he receives a discount of 10% on the marked price. If instead he uses his credit card, he has to pay a surcharge of 5% on top of the marked price. He decides to buy a carpet with a marked price of £180.
 - (a) Calculate the difference between the cash price and the credit card price.

His friend, Salib, would like to buy a different carpet. Salib calculates that the difference between the cash price and the credit card price for his carpet is £42.30.
 - (b) Calculate the marked price of Salib's carpet.

2. In a pen there are twelve camels. Some are Arabians and the remainder are Bactrians. An Arabian can carry 130 kg, and a Bactrian can carry 145 kg. The twelve camels can carry 1680 kg in total. Use simultaneous equations to calculate how many Arabian camels are in the pen.

3. The diagram shows a square ABCD, and two circles. Find the ratio area of larger circle: area of smaller circle.



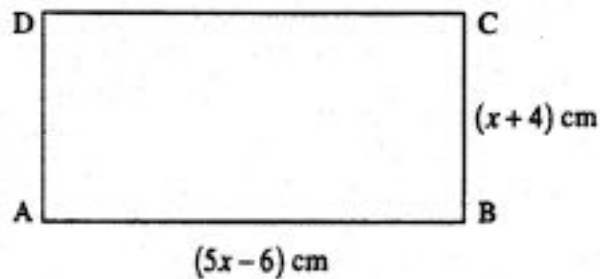
4. A man is planning a journey of 40 km with his donkey. If he rides his donkey, he can proceed at 6 kmh^{-1} . And if he walks beside his donkey, he can proceed at 10 kmh^{-1} . He would like to set out at 6.00 a.m. and reach his destination at 2.00 p.m. He will allow himself a break of 90 minutes during the journey.

Letting x km be the distance he rides his donkey, write down and solve an equation to find the value of x .

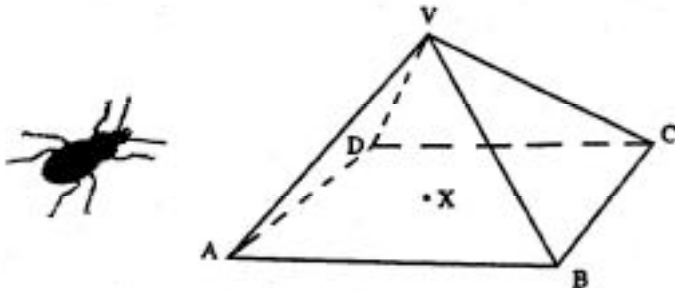
5. The diagram shows a rectangle, ABCD, in which $AB = DC = (5x - 6)$ cm, and $AD = BC = (x + 4)$ cm. Given that $AC = 26$ cm,

(a) show that $x^2 - 2x - 24 = 0$,

(b) find the value of x .



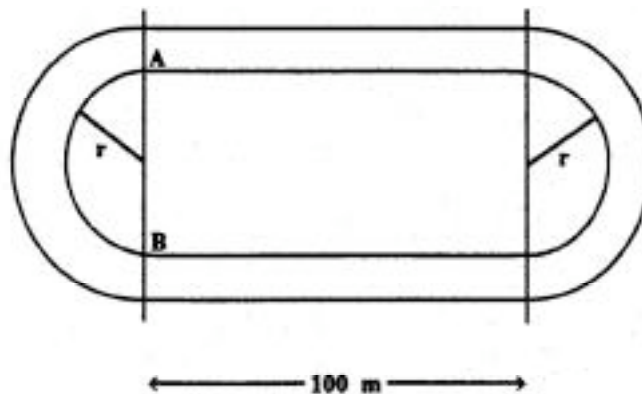
6. This year the Radley College Scholarship Beetle is tackling a pyramid.



The pyramid has a square base ABCD, of side 60 m. The vertex of the pyramid, V, is directly above X, the centre of the base, and $\angle AVC = 90^\circ$. The beetle is at A, and would like to get to C. Find how far it goes if it:

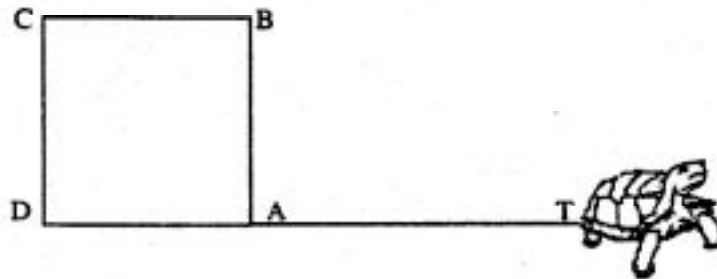
- goes along the edge from A to B and then along the edge from B to C,
- burrows directly from A to C,
- goes along the edge from A to V and then down the edge from V to C,
- goes along the edge from A to V, then burrows straight down from V to X, and then straight from X to C.

7.



- Given that the length, AB, of the inside of one of the semicircular bends on the running track shown is 100m, calculate the radius r , giving your answer to 3 significant figures.
- If one person runs a complete lap of the track at a constant distance of 2m from the inside and a second person runs a lap at a constant distance of 5m from the inside, calculate how much further the second person will have run if they were not given stagger starts. Give your answer to one decimal place.
- If the inner 2m wide strip of the complete track has to be resurfaced, calculate the area of material required. Give your answer to the nearest 10m^2 .

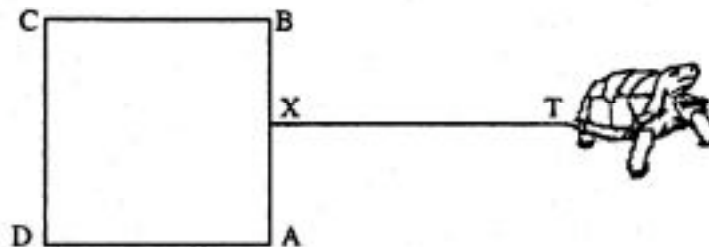
8. The Radley College Scholarship Tortoise is tethered to the corner, A, of a square box, as shown below. The square has sides of length 3m.



As usual the box is placed on a large lawn. The length of the string between A and T (the tortoise) is 12m.

- a) Initially D, A and T are in a straight line, with the string taut. The tortoise sets off in an anti-clockwise direction, keeping the string taut. How far does he walk before bumping into the box at A?

If, instead, the string is fixed to a point X on AB as in the diagram below, when $\angle BXT = 90^\circ$, and again the tortoise walks in an anti-clockwise direction keeping the string taut,



- b) find the position of X, given that he walks $\left(\frac{31\pi}{2}\right)$ metres before reaching X again.

9. A new operation, $*$, is defined by $a * b = a^2 + ab - b$.
So, for example, $3 * 2 = 3^2 + 3 \times 2 - 2 = 13$.

- (a) Work out $5 * 3$
- (b) Solve the equation $4 * x = 5$
- (c) Solve the equation $x * 1 = 5$
- (d) Prove that $x * (-x) = x$

10. The mayor of Dahab is about to introduce a new phone system. He decides that each phone number should have four digits, each digit being 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. So, possible phone numbers are 0000, 0001, 0002, etc.

- a) how many phone numbers are possible?
- b) how many phone numbers have all the digits different?

At a council meeting it is decided that no number should begin with 0.

- c) how many phone numbers are now possible?
- d) how many of these numbers have all the digits different?

11. The formula for the sum of the squares of the integers between 1 and n is

$$\frac{n}{6} (n+1) (2n+1)$$

$$\text{i.e. } 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n}{6} (n+1)(2n+1)$$

$$\text{So, for example, } 1^2 + 2^2 + 3^2 + 4^2 = \frac{4}{6} (4+1)(8+1) = 30.$$

Use this formula to find the value of each of the following:

- a) $1^2 + 2^2 + 3^2 + \dots + 50^2$
- b) $1^2 + 2^2 + 3^2 + \dots + 100^2$
- c) $51^2 + 52^2 + 53^2 + \dots + 100^2$
- d) $2^2 + 4^2 + 6^2 + \dots + 100^2$
- e) $1^2 + 3^2 + 5^2 + \dots + 99^2$

12. a) XY and YX are two digit numbers (X and Y are between 0 and 9 inclusive). Work out all the possible values X and Y could have if the following subtraction sum is true.

$$\begin{array}{r} XY \\ - YX \\ \hline 72 \end{array}$$

b) ANNA and NAAN are four digit numbers (as before, A and N are between 0 and 9 inclusive). Find the possible solutions if the following subtraction sum is true:

$$\begin{array}{r} ANNA \\ - NAAN \\ \hline 2673 \end{array}$$